Conditional Statements

Objective

Identify, write, and analyze the truth value of conditional statements.

Write the inverse, converse, and contrapositive of a conditional statement.

Lesson Presentation

Lesson Review
Objectives

Identify, write, and analyze the truth value of conditional statements.

Write the inverse, converse, and contrapositive of a conditional statement.
<table>
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<th>Vocabulary</th>
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<td>conditional statement</td>
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<td>hypothesis</td>
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<td>conclusion</td>
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<td>truth value</td>
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<td>negation</td>
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<td>converse</td>
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<tr>
<td>contrapositive</td>
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<td>logically equivalent statements</td>
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By phrasing a conjecture as an if-then statement, you can quickly identify its hypothesis and conclusion.
Example 1: Identifying the Parts of a Conditional Statement

Identify the hypothesis and conclusion of each conditional.

A. If today is Thanksgiving Day, then today is Thursday.
   
   Hypothesis: Today is Thanksgiving Day.
   
   Conclusion: Today is Thursday.

B. A number is a rational number if it is an integer.
   
   Hypothesis: A number is an integer.
   
   Conclusion: The number is a rational number.
Identify the hypothesis and conclusion of the statement.

"A number is divisible by 3 if it is divisible by 6."

Hypothesis: A number is divisible by 6.
Conclusion: A number is divisible by 3.
"If $p$, then $q$" can also be written as "if $p$, $q$," "$q$, if $p$," "$p$ implies $q$," and "$p$ only if $q$.”

Many sentences without the words *if* and *then* can be written as conditionals. To do so, identify the sentence’s hypothesis and conclusion by figuring out which part of the statement depends on the other.
Example 2A: Writing a Conditional Statement

Write a conditional statement from the following.

An obtuse triangle has exactly one obtuse angle.

Identify the hypothesis and the conclusion.

An obtuse triangle has exactly one obtuse angle.

If a triangle is obtuse, then it has exactly one obtuse angle.
Write a conditional statement from the following.

If an animal is a blue jay, then it is a bird.

The inner oval represents the hypothesis, and the outer oval represents the conclusion.
Write a conditional statement from the sentence “Two angles that are complementary are acute.”

Two angles that are complementary are acute.

If two angles are complementary, then they are acute.
A conditional statement has a **truth value** of either true (T) or false (F). It is false only when the hypothesis is true and the conclusion is false.

To show that a conditional statement is false, you need to find only one counterexample where the hypothesis is true and the conclusion is false.
Example 3A: Analyzing the Truth Value of a Conditional Statement

Determine if the conditional is true. If false, give a counterexample.

If this month is August, then next month is September.

When the hypothesis is true, the conclusion is also true because September follows August. So the conditional is true.
Determine if the conditional is true. If false, give a counterexample.

If two angles are acute, then they are congruent.

You can have acute angles with measures of $80^\circ$ and $30^\circ$. In this case, the hypothesis is true, but the conclusion is false.

Since you can find a counterexample, the conditional is false.
Example 3C: Analyzing the Truth Value of a Conditional Statement

Determine if the conditional is true. If false, give a counterexample.

If an even number greater than 2 is prime, then $5 + 4 = 8$.

An even number greater than 2 will never be prime, so the hypothesis is false. $5 + 4$ is not equal to 8, so the conclusion is false. However, the conditional is true because the hypothesis is false.
Determine if the conditional “If a number is odd, then it is divisible by 3” is true. If false, give a counterexample.

An example of an odd number is 7. It is not divisible by 3. In this case, the hypothesis is true, but the conclusion is false. Since you can find a counterexample, the conditional is false.
Remember!

If the hypothesis is false, the conditional statement is true, regardless of the truth value of the conclusion.

The **negation** of statement $p$ is “not $p$,” written as $\sim p$. The negation of a true statement is false, and the negation of a false statement is true.
### Related Conditionals

<table>
<thead>
<tr>
<th>Definition</th>
<th>Symbols</th>
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<tbody>
<tr>
<td>A conditional is a statement that can be written in the form &quot;If $p$, then $q$.&quot;</td>
<td>$p \rightarrow q$</td>
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<td>The <strong>converse</strong> is the statement formed by exchanging the hypothesis and conclusion.</td>
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<td>The <strong>inverse</strong> is the statement formed by negating the hypothesis and conclusion.</td>
<td>$\sim p \rightarrow \sim q$</td>
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<tr>
<td>The <strong>contrapositive</strong> is the statement formed by both exchanging and negating the hypothesis and conclusion.</td>
<td>$\sim q \rightarrow \sim p$</td>
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Write the converse, inverse, and contrapositive of the conditional statement. Use the Science Fact to find the truth value of each.

If an animal is an adult insect, then it has six legs.

Science Fact
Adult insects have six legs.
No other animals have six legs.
Example 4: Biology Application

If an animal is an adult insect, then it has six legs.

Converse: If an animal has six legs, then it is an adult insect.

No other animals have six legs so the converse is true.

Inverse: If an animal is not an adult insect, then it does not have six legs.

No other animals have six legs so the inverse is true.

Contrapositive: If an animal does not have six legs, then it is not an adult insect.

Adult insects must have six legs. So the contrapositive is true.
Related conditional statements that have the same truth value are called **logically equivalent statements**. A conditional and its contrapositive are logically equivalent, and so are the converse and inverse.

**Helpful Hint**

The logical equivalence of a conditional and its contrapositive is known as the Law of Contrapositive.
Identify the hypothesis and conclusion of each conditional.

1. A triangle with one right angle is a right triangle.
   H: A triangle has one right angle.
   C: The triangle is a right triangle.

2. All even numbers are divisible by 2.
   H: A number is even.
   C: The number is divisible by 2.

3. Determine if the statement “If $n^2 = 144$, then $n = 12$” is true. If false, give a counterexample.
   False; $n = -12$. 
Lesson Review: Part II

Identify the hypothesis and conclusion of each conditional.

4. Write the converse, inverse, and contrapositive of the conditional statement “If Maria’s birthday is February 29, then she was born in a leap year.” Find the truth value of each.

**Converse:** If Maria was born in a leap year, then her birthday is February 29; False.

**Inverse:** If Maria’s birthday is not February 29, then she was not born in a leap year; False.

**Contrapositive:** If Maria was not born in a leap year, then her birthday is not February 29; True.